# INTRODUCTION TO PROBABILITY

The goal of this lecture is to become familiar with the basics of probability theory and to understand relationships of probabilities

# Objectives

- After this lecture you should be able to:
  - Describe uncertainty using probability terms
  - Determine how likely an event is to occur
  - Combine information regarding more than one event
  - Solve probability problems for use in decision making

# Introduction

- Probabilities are useful for answering questions
  - What is the "chance" that sales will decrease if the price of the product is increased?
  - What is the "likelihood" that the new assembly method will increase productivity?
  - How "likely" is it that the project will be completed on time?
  - What are the "odds" that the new investment will be profitable given that corn prices are low?

# Analyzing Uncertainty

- Random experiments
  - A process or course of action that results in one of a number of possible **outcomes**
  - The **sample space** is the set of all possible experimental **outcomes**
  - Outcomes must be **mutually exclusive** and **collectively exhaustive**
  - Any one particular event is referred to as an **outcome** from the experiment

# Random Experiment - Tossing a Coin

- The events are Heads & Tails
  - These are the only two possibilities (**collectively exhaustive**)
  - The result must either be heads or tails and cannot be both (**mutually exclusive**)
  - An **outcome would** be Heads or Tails

# Probability of an Event

Relative frequency, Theoretical, and Subjective Probabilities

# How likely is an Event?

- Probability of an Event
  - A number between 0 (never happens) and 1 (always happens) (often expressed as a percentage)
    - The likelihood of occurrence of an event
  - Each random experiment has many probability numbers
    - One probability number for each event

# Farm Income Ranges Examples

**Distribution of farms, total production, and assets by farm type, 2007**

| Farm type | Farms | Value of production | Farm assets |
|---|---|---|---|
| | | *Percent of U.S. total* | |
| Small family farms:[1] | | | |
| Retirement | 18.4 | 1.6 | 12.9 |
| Residential/lifestyle | 45.1 | 4.2 | 26.0 |
| Farming-occupation | | | |
| Low-sales | 19.8 | 4.0 | 17.3 |
| Medium-sales | 5.1 | 6.6 | 7.9 |
| Large-scale family farms:[1] | | | |
| Large family farms | 4.3 | [illegible] | 9.3 |
| Very large family farms | 5.0 | 53.7 | 20.1 |
| Nonfamily farms[1,2] | 2.4 | 17.7 | 6.6 |

[1]Small farms have sales less than $250,000; large-scale farms have sales of $250,000 or more; no sales limit for nonfamily farms.
[2]Nonfamily farms include any farm where the majority of the business is not owned by the operator and individuals related to the operator.

Source: USDA, National Agricultural Statistics Service and Economic Research Service, 2007 Agricultural Resource Management Survey, Phase III.

# Assigning Probabilities

- Two General Rules for Probabilities
  - The probability assigned to each sample point must be between 0 and 1(often expressed as a percentage)
  - The sum of all of the experimental outcomes' probabilities must be 1

# Approaches to Assigning Probabilities

- Relative frequency
- Subjective
- Theoretical

# Relative Frequency Method

- Expresses an outcome's probability as the <u>long-run</u> relative frequency of occurrence
- This is also known as the **empirical probability**
- Using the **Law of Large Numbers**
  - Relative frequency is good "best guess"
  - The larger the sample the better

# Relative Frequency Method

**Table 13—Estimated seed sales and shares of U.S. market for major field crops, 1997**

| Company | Total sales | Total market share[1] | Corn market share | Soybean market share | Cotton market share |
|---|---|---|---|---|---|
| | *Million dollars* | *Percent* | | | |
| Pioneer Hi-Bred | 1,178 | 33.6 | 42 | 19 | 0 |
| Monsanto[2] | 541 | 15.4 | 14 | 19 | 11 |
| Novartis | 262 | 7.5 | 9 | 5 | 0 |
| Delta & Pine Land[3] | 79 | 2.3 | 0 | 0 | 73 |
| Dow Agrosciences / Mycogen | 136 | 3.9 | 4 | 4 | 0 |
| Golden Harvest | 93 | 2.6 | 4 | 0 | 0 |
| AgrEvo/Cargill | 93 | 2.6 | 4 | 0 | 0 |
| Others | 1,121 | 32.0 | 23 | 53 | 16 |
| Total | 3,503 | 100.0 | 100 | 100 | 100 |

[1] Total market shares in this table include only corn, soybeans, and cotton.
[2] Monsanto acquired Dekalb in 1997 and Asgrow in 1998.
[3] The merger between Monsanto and Delta & Pine Land in 1998 was called off in December 1999.

*Sources:* Market shares for corn and soybeans: Hayenga (1998); cotton: USDA, AMS. Total crop sales calculated from acreages and seed cost per acre: USDA's Agricultural Resource Management Survey data (1998) and *Agricultural Statistics* (USDA, 1998).

**Econ Reading: The Seed Industry in US Agriculture: An Exploration of Data**

**Pros**

Gives a starting point

Might strongly correlate to short term decisions

**Cons**

History might not be best guide of long-run future

Innovative products

# Subjective Method

- In many real world cases
  - The outcomes are not equally likely
  - There is no historical data
  - There is no experimental data
- An individual assigns a probability to an outcome
  - Degree of belief that an experimental outcome will occur
  - Subjective assessments can be tested for sensitivities

# Subjective Method

- What probabilities are associated with potential farm economy outcomes?
  - Agricultural economy growth – 30%
  - Agricultural economy stable – 50%
  - Agricultural economy decline – 20%

- Options need to be mutually exclusive and collectively exhaustive

# Theoretical Method

- An experiment has "n" possible outcomes
  - Each outcome is equally likely
  - Probability of one outcome's occurrence is 1/n
- The coin flip example
  - Coin landing heads is as equally likely as landing tails
  - "n" = 2 (heads or tails)
  - Probability of heads is ½
  - Probability of tails is also ½

# Theoretical Method

| Roll | Expected |
|---|---|
| 2 | 2.78% (1/36) |
| 3 | 5.56% (2/36) |
| 4 | 8.33% (3/36) |
| 5 | 11.11% (4/36) |
| 6 | 13.89% (5/36) |
| 7 | 16.67% (6/36) |
| 8 | 13.89% (5/36) |
| 9 | 11.11% (4/36) |
| 10 | 8.33% (3/36) |
| 11 | 5.56% (2/36) |
| 12 | 2.78% (1/36) |

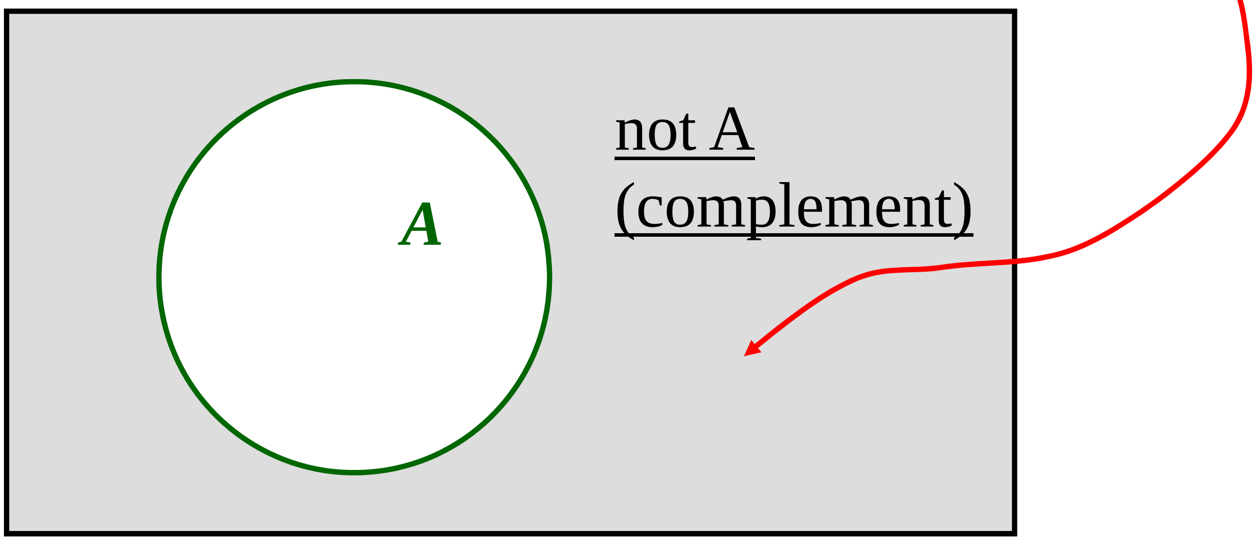
# Venn Diagrams and Conditional Probabilities

# Combining Events

- **Complement** of the event A
  - Happens whenever A does not happen
- **Union** of events A and B
  - Happens whenever either A or B or **both** events happen
- **Intersection** of A and B
  - Happens whenever both A and B happen
- **Conditional probability** of A given B
  - The updated probability of A, possibly changed to reflect the fact that B has happened

# Complement of an Event

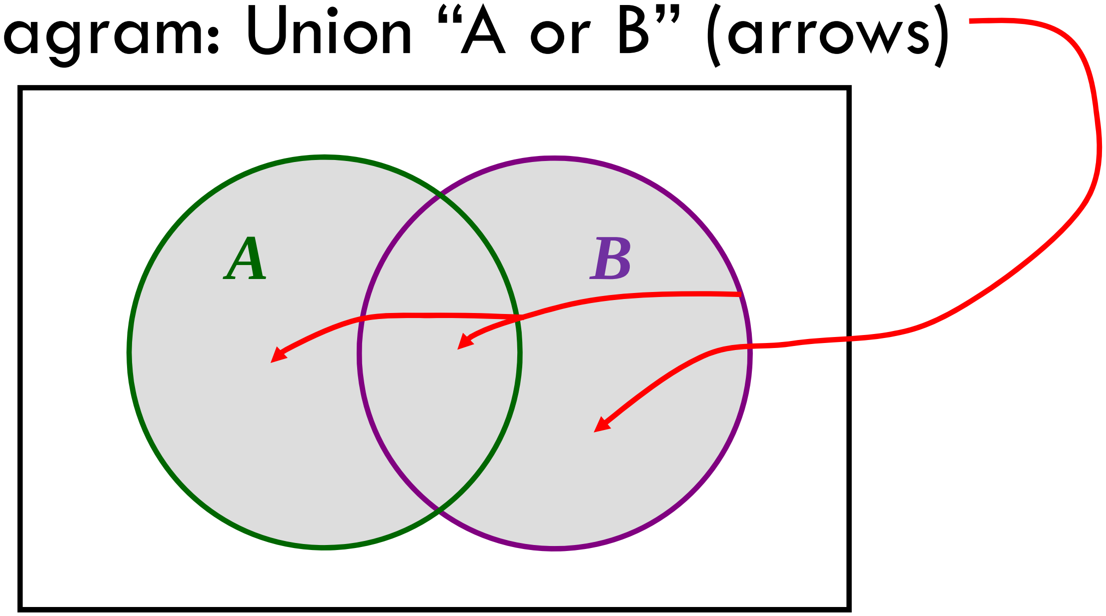
- The event "not A" happens whenever A does not
- Venn diagram: A (in circle), "not A" (shaded)




- Probability(not A) = 1 – Prob(A)
  - If Prob(Succeed) = 0.7, then Prob(Fail) = 1–0.7 = 0.3

# Union of Two Events

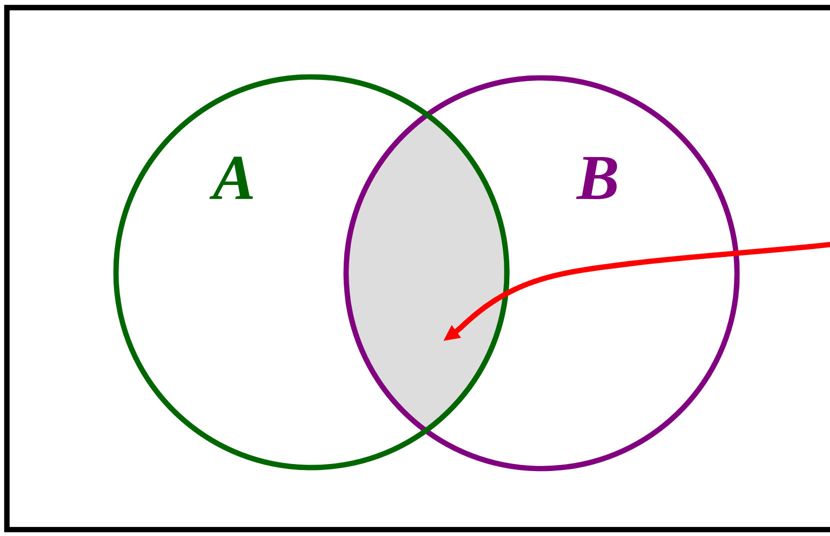
- Union happens whenever either (or both) happen
- Venn diagram: Union "A or B" (arrows)




  - e.g., A = "get Intel job offer", B = "get GM job offer"
    - Did the union happen? Congratulations! You have a job
  - e.g., Did I have eggs or cereal for breakfast? "Yes"

# Intersection of Two Events

- Intersection happens whenever both events happen
- Venn diagram: Intersection "A and B" (arrow)

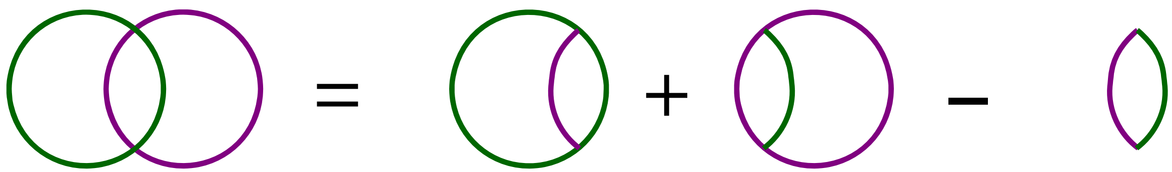



- e.g., A = "sign contract", B = "get financing"
  - Did the intersection happen? Project has been launched!
  - e.g., Did I have eggs and cereal for breakfast? "No"

# Twitter is the intersection

# Relationship Between and and or

- Prob(A or B) = Prob(A)+Prob(B)–Prob(A and B)



- Prob(A and B) = Prob(A)+Prob(B)–Prob(A or B)

# Farm location and type

- Prob(Fruitful Rim) = 0.119
- Prob(Residential/lifestyle) = 0.451
- Prob(Fruitful Rim and Residential/Lifestyle) = 0.048

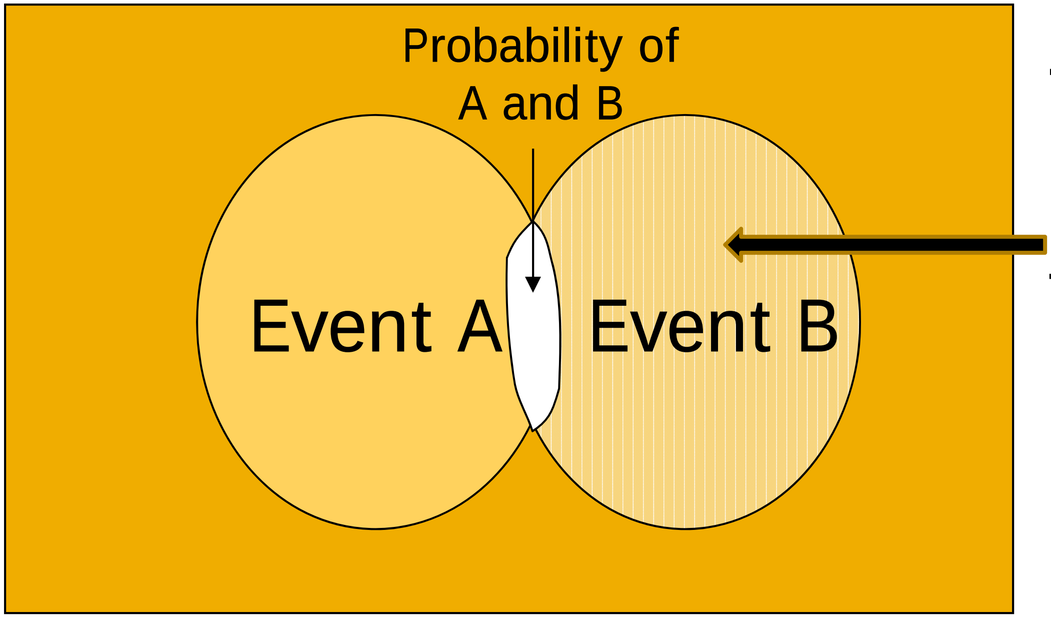
- Then we must have
  - Prob(Fruitful Rim or Residential/Lifestyle) = 0.119+0.451–0.048 = 0.522

# The Conditional Probability

- In many probability situations, being able to determine the probability of one event when another event is known to have occurred is important
- Suppose that we have an event A with probability P(A) and that we obtain new information or learn that another event B has occurred
- If A is related to B we want to take advantage of this information

# The Conditional Probability

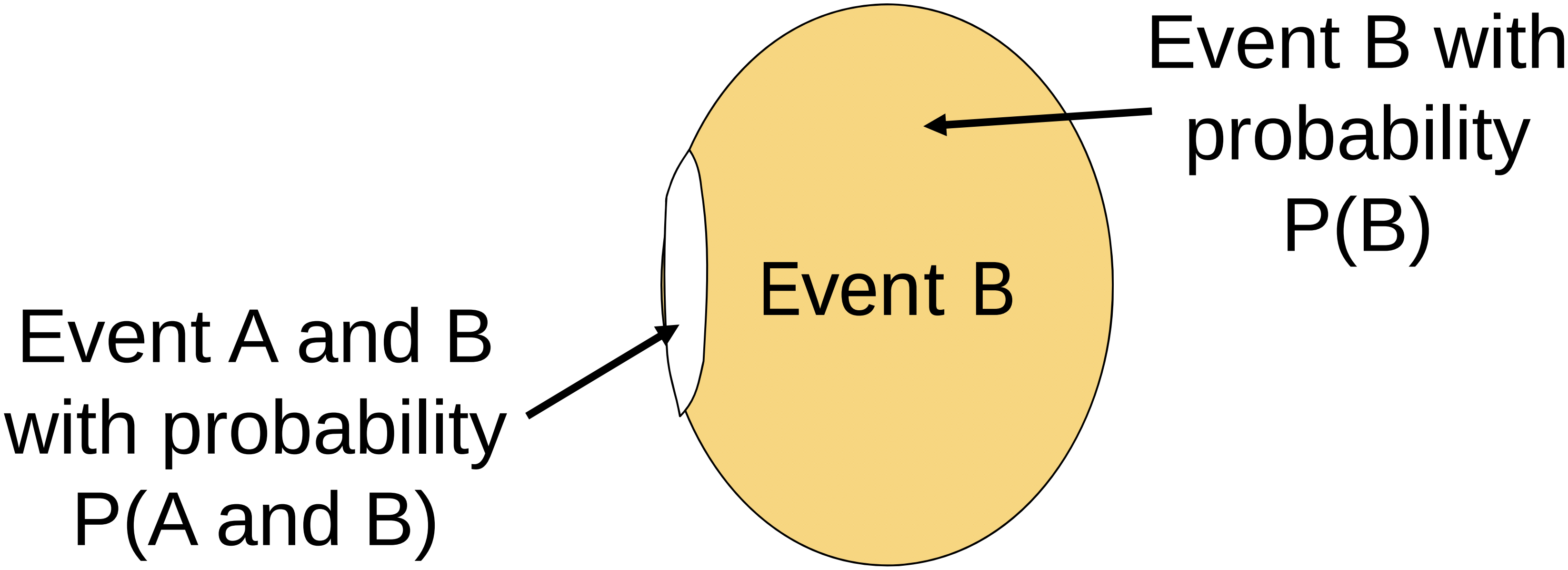
- The new probability of event A is written P(A|B) which is read as the probability of A given the condition that B has occurred
- The general definition is:




The light shaded region denotes that B has occurred

# The Conditional Probability

**<u>Once Event B has occurred</u>**




$$Thus\ P(A|B) = \frac{P(A\ and\ B)}{P(B)}$$

# Seed Adoption

- Illinois Farmers
  - Prob(grows corn) = 0.70, Prob(early adopter) = 0.135, Prob(grows corn and early adopter) = 0.07
- Conditional probability of early adopter given that they grow corn
  - P(early adopter | grows corn) = Prob (early adopter and grows corn)/Prob (grows corn) = 0.07/0.70 = .10
  - 10% of corn farmers are early adopters

# Seed Adoption

- Illinois Farmers
  - Prob(grows corn) = 0.70, Prob(early adopter) = 0.135, Prob(grows corn and early adopter) = 0.07
- Conditional probability of grows corn given that they are early adopters
  - P(grows corn | early adopter) = Prob (early adopter and grows corn)/Prob (early adopter) = 0.07/0.135 = 0.519
  - About 52% of early adopters (among Illinois farmers) grow corn

# Independent and Mutually Exclusive Events

- Do not confuse mutually exclusive events with independent events
- Two events with nonzero probabilities cannot be both mutually exclusive and independent
- If one mutually exclusive event is known to occur, the probability of the other occurring is zero
  - Thus, they cannot be independent

# Method for Solving Probability Problems

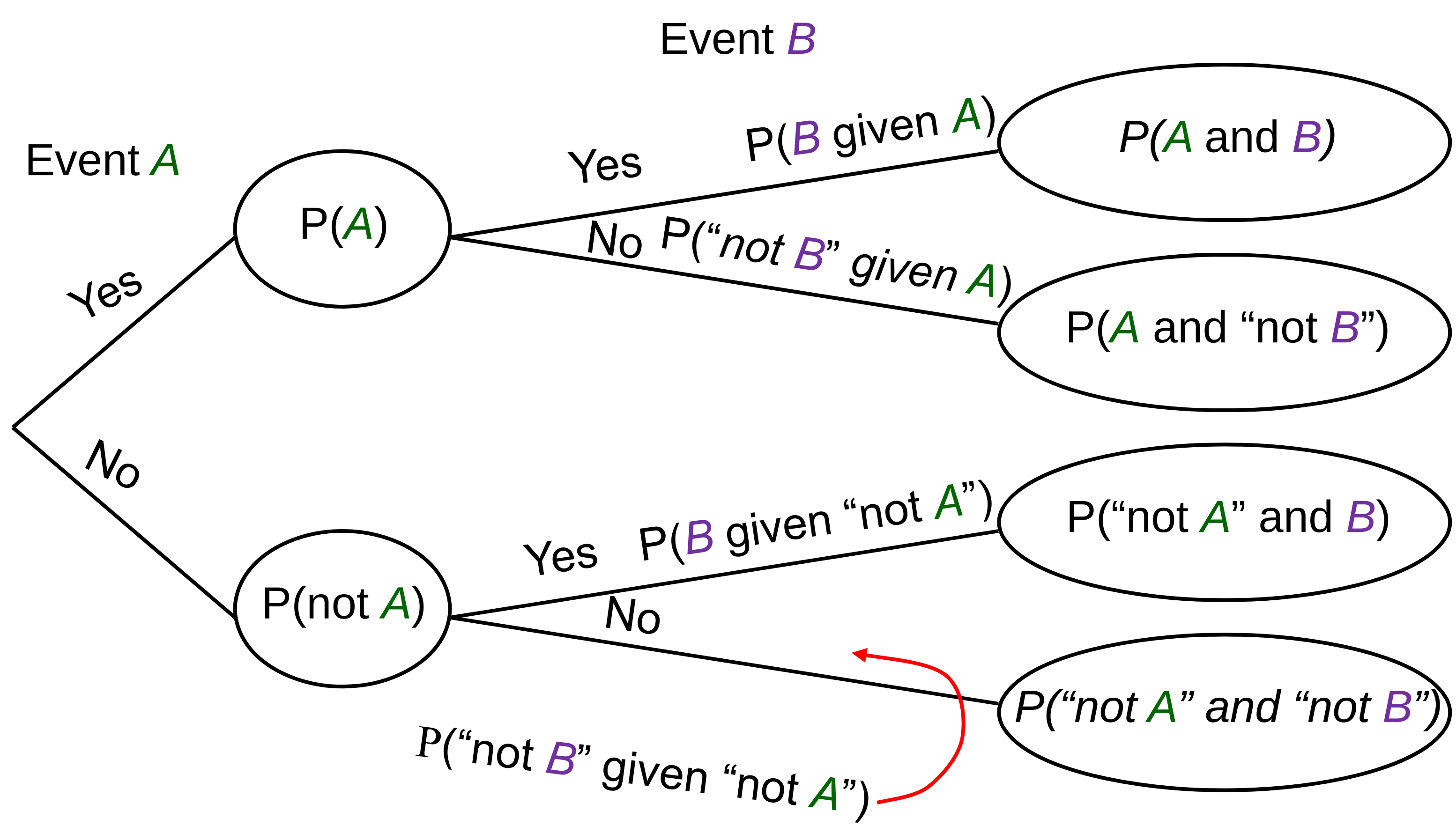
- Given probabilities for some events (perhaps union, intersection, or conditional)
  - Find probabilities for other events
- Use a probability tree

# Probability Tree

- Record the basic information on the tree
- Usually three probability numbers are given
- The tree helps guide your calculations
  - Each column of circled probabilities adds up to 1
  - Circled probability times conditional probability gives next probability
  - For each group of branches
    - Conditional probabilities add up to 1
    - Circled probabilities at end add up to probability at start

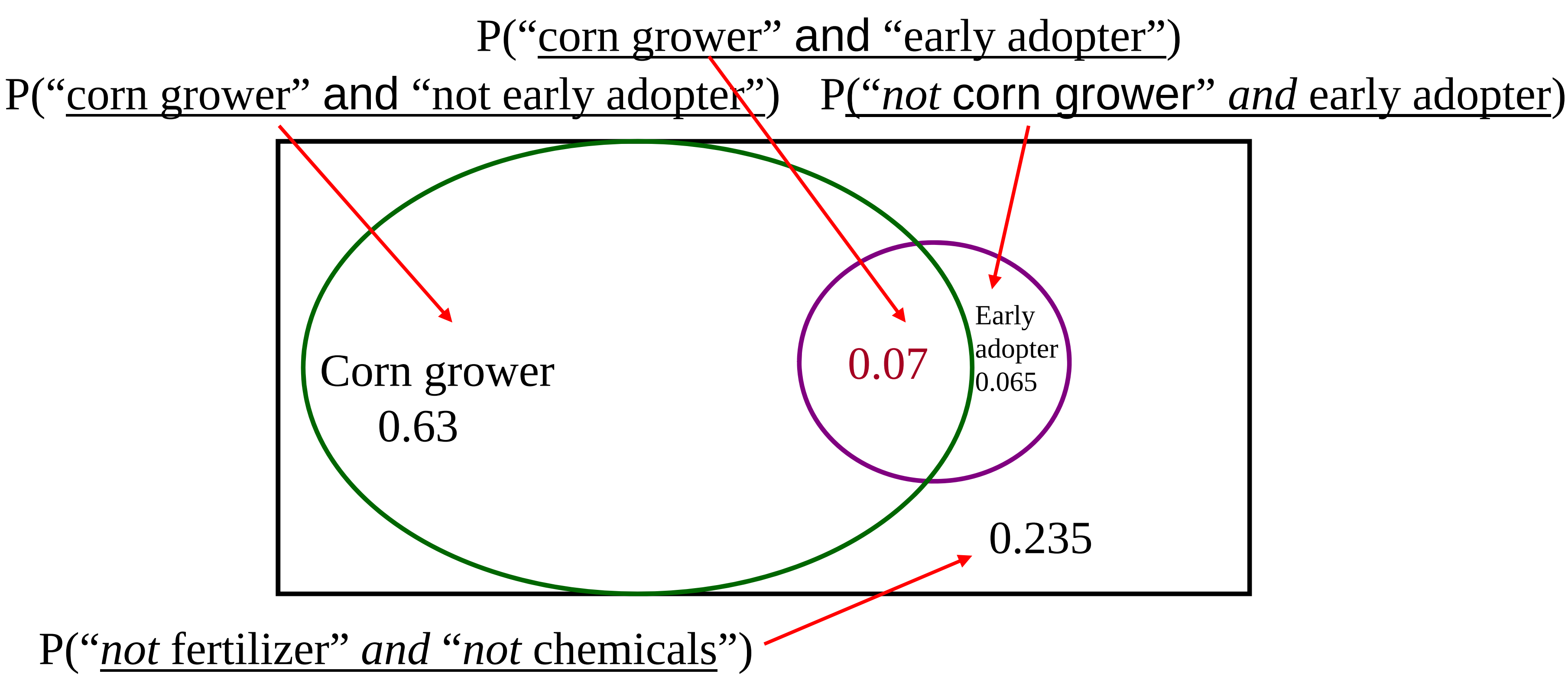
# Probability Tree

Shows probabilities and conditional probabilities



# Venn Diagram

- Venn diagram probabilities correspond to right-hand endpoints of probability tree



# Joint Probability Table

- Shows probabilities for each event, their complements, and combinations using ***and***
- Note: rows add up, and columns add up

P("not corn grower" and early adopter)

*P(Early Adopter and Corn Grower)*

P(early adopter)

P(*not* early adopter)

| | | Corn Grower | | |
|---|---|---|---|---|
| | | Yes | No | |
| Early Adopter | Yes | 0.07 | 0.065 | 0.135 |
| | No | 0.63 | 0.235 | 0.865 |
| | | 0.70 | 0.30 | 1 |

P(corn grower *and* "*not* early adopter")

*P*("not corn grower" and "not early adopter")

P(corn grower)

# Summary

- You should now have a good understanding of
  - Experiments
  - Assigning Probabilities
  - Probability Rules
- Review these concepts as often as necessary to become familiar with them
- Assignment 2 will give you another opportunity to review and apply this material